

# A Statistical Analysis of Some Mechanical Properties of Manila Rope

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Breaking strength, weight, and circumference are three important qualities that are determined when fiber ropes are submitted for test.

These properties are all subject to some variation because of differences in fiber quality, method of fabrication, and previous treatment. The results of tests on more than 800 samples of 3-strand manila ropes ranging in size from  $\frac{3}{16}$  in. to 3 in. in diameter are tabulated and analyzed by the methods of modern mathematical statistics. Considerable emphasis is placed on the rationale and details of the methods of analysis, as they are considered to be applicable to a broad variety of similar projects.

## I. Introduction

Strength, weight, and size are usually determined for samples of fiber rope submitted to the Bureau for acceptance tests. These properties have been found to be subject to some variation in manila rope by Stang and Strickenberg [1].<sup>1</sup> This variation would appear to be caused by differences in fiber quality and in the mode of fabrication of the rope.

In many applications of fiber rope where even small economies in weight are important, i. e., in use of ropes on cargo airplanes, a knowledge of the probable strength of a set of ropes can result in an increase of the pay load. For other applications where ropes pass over sheaves and through eyes, the probable range of size is an important design consideration.

The manila ropes discussed in this paper were submitted for test by a Government agency from 1938 to 1941. They represent material supplied by rope works and rope contractors in the 3 years just prior to the loss of sources of manila fiber in World War II. This accounts for the heterogeneous sizes of the samples available for the various nominal diameters, as the smaller sizes and the integral multiples of  $\frac{1}{4}$  in. in diameter are ordered more frequently than other sizes. Although some

data were available for 4-strand manila rope, they were not of sufficient quantity to allow statistical treatment and consequently are not discussed here. In general, the requirements for 4-strand rope in the Federal Specification [2] will yield a basis for comparison with the 3-strand ropes treated in this paper.

The methods of test described are those found in the Federal Specification for manila rope [2], but the data are applicable to many situations where the conditions may be somewhat different. Whittemore [3] found that the speed of the moving head of the testing machine between 1 and 4 in./min had little effect on the observed strength of the rope. It has also been noted [4] that measurements of circumference with increasing loads on the specimen yielded decreasing changes in circumference for equal increments of load up to loads equal in pounds to 300 times the diameter in inches squared.

## II. Methods of Test

### 1. Circumference and Weight

Both weight and circumference of manila ropes were determined on an unspliced sample that had been conditioned in an atmosphere of  $65 \pm 2$  percent relative humidity and  $70^\circ \pm 2^\circ$  F for at least 72 hours preceding the test. The samples were

<sup>1</sup> Figures in brackets indicate the literature references at the end of this paper.

long enough to provide the minimum free length specified in table 1.

TABLE 1<sup>a</sup>.—Length of weight—circumference specimens

Circumference	Minimum free length
<i>In.</i>	<i>Ft.</i>
Below 3.....	10
3 to 6, incl.....	5
Over 6.....	3

<sup>a</sup> Values from Table III of Federal Specification for Rope, Manila.

The rope was placed in a horizontal testing machine and a tensile load  $P=200 D^2$  lb (where  $D$  is the nominal diameter of the rope in inches) was applied. A single fiber was passed snugly around the rope, near the middle of the sample, and cut where it overlapped. The cut length of fiber was measured and the circumference recorded to the nearest  $\frac{1}{16}$  in.

With the load,  $P$ , still applied to the rope, a length as specified in table 1 was marked off on the free length, and the load was then removed. The marked length was cut from the sample and weighed, and the weight in pounds per foot was computed.

## 2. Breaking Strength

A breaking strength sample had an eye splice at each end and measured from 5 to 6 ft between the inner ends of the splices. The samples were conditioned in the same fashion as the circumference-weight samples. The ropes were then removed from the conditioning room and the splices were immersed in water for 15 minutes in order to minimize the possibility of a failure in the splice.

Ropes with breaking strengths over 2,000 lb were loaded to failure in a horizontal hydraulic testing machine, and all others were tested using a horizontal, pendulum, screw testing machine. The ropes were loaded by means of 3-in. diameter steel pins passed through the eyes at each end of the sample. The speed of the moving head of the testing machine was 3 in./min during the tests. The tensile load was increased, until at the maximum load the rope failed in one or more strands.

## III. Methods of Statistical Analysis

Four fundamental relationships were studied: (1) Circumference as a function of nominal diameter, (2) weight as a function of nominal diameter, (3) strength as a function of nominal diameter, and (4) strength as a function of weight. The first three are useful in preparing specifications and predicting the characteristics of individual ropes purchased under a contract or order. It was therefore considered desirable in these cases to furnish careful predictions of the dispersions encountered in practice, as well as of the mean values, so that realistic tolerances for individual ropes can be set up. The fourth relationship is important as a measure of innate characteristics of the material. Central tendency, rather than the dispersion, seemed to be of paramount interest in this case.

Observations on 863 ropes were available for this study. With truly random sampling and a clearly defined and homogenous universe or population, such a large sample, if properly handled, should provide close estimates of the underlying frequency distributions and relationships of the variables. In such circumstances the use of empirical equations containing, if necessary, several parameters would be justified. In the present instance, no direct control could be exercised over the sampling, and the universe, which presumably consists of the entire National output of manila rope of the relevant nominal sizes during the period from 1938 to 1941, was far from homogeneous. The data, as might be expected, contained certain anomalies that are more or less directly attributable to the composite nature of this universe, or to the nonrandom sampling.

In spite of these shortcomings, the observations appeared to exhibit a sufficient amount of internal consistency to warrant the use of mathematical methods, provided that the description of the underlying causal situation attempted thereby should not be too elaborate.

The mean values of the dependent variable were represented in each case by a curve of the general form

$$\bar{Y}=kX^b, \quad (1)$$

where  $\bar{Y}$  designates the (arithmetic) mean value of the dependent variable and  $X$  the independent variable. The parameters  $k$  and  $b$  were adjusted to the data, except in the case of the circumference-diameter relationship, where  $b$  was arbitrarily taken as unity.

The curves were fitted by the following general process: All data were first transformed to logarithms (to the base ten), and then a regression equation of the form

$$\log Y = a + b \log X, \quad (2)$$

was fitted by the standard unweighted least squares method to the logarithmic data. At the same time the standard error of estimate  $s$  (that is, the root-mean-square deviation of the data about the regression line) was also determined. The resulting equation (2) yielded an optimum estimate of the mean  $\log Y$  for each  $\log X$ , provided that it can be assumed that the true relation between mean  $\log Y$  and  $\log X$  is exactly linear and that the standard deviation of the variable  $\log Y$  for a given  $\log X$  is a constant independent of  $X$ . As the antilog of the mean of a set of logarithms of  $Y$  is not the arithmetic mean of  $Y$  itself,<sup>2</sup> it was necessary to add a correction of some sort to (2) in transforming back to the form (1). The correction chosen in this case was  $1.15129s^2$ , which is based on the further assumption that  $\log Y$  is normally distributed for each value of  $\log X$  (see [5], pp. 120–121). Thus in terms of the notation used in connection with (2), formula (1) becomes

$$\bar{Y} = 10^{a + 1.15129s^2 X^b}. \quad (3)$$

The calculations involved in the curve-fitting process were carried out almost entirely on punched-card machinery at the Computation Laboratory of the Bureau. The logarithmic transformation was accomplished automatically by the use of master logarithm cards in conjunction with a collater that simultaneously punched both the logarithms and their squares onto the data cards. Cross products and cumulative sums were then obtained in the standard way on multiplying punches and tabulators.

A few general remarks on the underlying rationale of this method of curve fitting and the

choice of the type of curve are in order, as the situation about to be described is typical of that frequently encountered in tests of materials. The most elementary physical theory of the data would suggest that mean observed circumference should be directly proportional to the nominal diameter (which is actually defined to be one-third of the nominal circumference), and that observed weight and strength should then be directly proportional to the square of the observed circumference. This would ordinarily imply that the mean values of circumference, weight, and strength, as functions of nominal diameter, could be most appropriately estimated by fitting linear trends by the classical least squares method to the observed values, respectively, of circumference, the square root of weight, and the square root of strength.

Unfortunately, as is so often the case, the data for all of these variables reveal an unmistakable tendency for the dispersion of the readings (measured, say, by their standard deviation) to increase with their mean. This would necessitate the use of a weighted least squares solution. The simple unweighted least squares solution would give an inefficient estimate of the mean and would not yield any over-all estimates at all for the variances within nominal diameter groups. But the difficulty of determining *a posteriori* the proper individual weights to use in such cases has led in recent years to the widespread use of transformations of the scale, such as the logarithmic transformation here used, to effect, at least theoretically, a stabilization of variance over the whole range of values of the mean. Such transformations under certain circumstances have the additional property of rendering the distribution of the transformed variable more nearly normal or Gaussian.

In the present case, as far as could be determined from the data, the standard deviation of the observed circumference seemed to be roughly a linear function of the mean. The transformation ordinarily employed in that event is a logarithmic transformation [5]. Preliminary exploration revealed that this transformation indeed seemed to stabilize the variance of the circumference measurements. It also seemed to stabilize the variance of the weight and strength measurements. This would follow mathematically if the transformation really did stabilize the circumference variance

<sup>2</sup>It is the *geometric mean* of  $Y$ , however.

and if mean weight and strength really were directly proportional to some power of the circumference. Thus additional evidence was furnished thereby as to the correctness of the choice of the logarithmic transformation for the circumference data.

As for the choice of the curve, it is natural to choose a type that contains as few parameters as possible and that can easily be handled in conjunction with the logarithm transformation. Stang and Stickenberg [1] chose to use a one-parameter curve of the form

$$\bar{Y} = k X (X+1) = k X^2 + k X,$$

to represent the strength-diameter relationship. Although this equation is readily adaptable to the logarithmic least squares approach (it seems to have been fitted empirically in [1]), nevertheless the first degree term is not easy to explain in terms of the physical theory. Exponential equations of the type (1), on the other hand, not only can be conveniently handled after a logarithmic transformation, but also accord very well with a slight extension of the simplest physical theory of the tests. They imply that weights and strengths of a series of ropes of different sizes whose successive nominal diameters are in a constant ratio will themselves be in a constant ratio.

As a check on the adequacy of (1), three-parameter curves of the type

$$\bar{Y} = C_1 + C_2 X + C_3 X^2,$$

were actually fitted to the weight-diameter relations, and in spite of the additional parameter, the results obtained were approximately equivalent to those obtained with (1).

## IV. Results and Discussion

The various equations for mean values, obtained as described in section III, together with the corresponding values of the regression constants  $a$ ,  $b$ , and  $s$  associated with eq 2 of section III are all given in table 2.

In interpreting the equations, it should be understood that they give the estimated mean values of the dependent variable for each fixed, predetermined value of the independent variable. Thus the last equation, which gives the estimated mean of  $S$  as a function of  $W$ , cannot be obtained by merely eliminating  $D$  between the second and third equations, because the distribution of values of  $S$  corresponding to a fixed  $D$ , say  $D=D_0$  and the distribution of values of  $S$  corresponding to a fixed  $W$ , are not in general quite the same, even if the fixed  $W$  was determined by setting  $D=D_0$  in the second equation. (Actually the two methods of deriving the fourth equation in table 2 happen to agree to two significant figures in the exponent and three in the constant factor, owing to the relatively small scatter of the data about the various curves.)

The standard errors  $s_a$  and  $s_b$  of  $a$  and  $b$  are also presented in table 2. It will be noted that these standard errors turn out to be exceedingly small. This is partly due to the large number of items entering into the sample, and partly due to the relatively small size of  $s$  in each case. Statistical theory would state that if the sampling had been truly random and if curves of the type used exactly described the mean relationships in the sampled universe, then ranges of  $a \pm 3s_a$  and  $b \pm 3s_b$  would, with high probability, contain the corresponding "true" values of  $a$  and  $b$ ; that is, the values for the universe. Due mainly to the

TABLE 2.—Regression equations and statistics of distributions of circumferences, weights, and strengths of 3-strand manila rope

Dependent variable			Independent variable			Equation of means	Regression statistics <sup>a</sup>				
Meaning	Sym- bol	Unit	Meaning	Sym- bol	Unit		$a$	$b$	$s$	$s_a$	$s_b$
Circumference.....	$C$	1/16 in.....	Nominal diameter.....	$D$	1/16 in.....	$C = 3.119 D$ .....	0.49353	(b)	0.02026	0.00069	-----
Weight.....	$W$	lb/ft.....	Nominal diameter.....	$D$	1/16 in.....	$W = 0.001447 D^{1.4527}$ .....	-2.84023	1.88268	.02541	.0038	0.0033
Strength.....	$S$	lb.....	Nominal diameter.....	$D$	1/16 in.....	$S = 70.481 D^{1.82819}$ .....	1.845979	1.828193	.042619	.0064	.0055
Strength.....	$S$	lb.....	Weight.....	$W$	lb/ft.....	$S = 40278 W^{0.96894}$ .....	4.602511	0.968940	.047059	.0028	.0032

<sup>a</sup> For explanation of  $a$ ,  $b$ , and  $s$ , see discussion of eq. 2 in sec. III. The symbols  $s_a$  and  $s_b$  denote the standard errors of  $a$  and  $b$ , computed under the assumption that for each value of  $D$ , the data constitute a random sample from a universe of such data. (See sec. IV.)

<sup>b</sup> Not adjusted to data.



fact that the method of sampling was not rigorously random, such a statistical interpretation of  $s_a$  and  $s_b$  is apparently unwarranted. It is believed, however, that these standard errors afford ample basis for the following important general statements about the coefficients and exponents in the equations in table 2:

(1) In the case of the circumference-diameter relationship, the coefficient of  $D$  is significantly larger than 3, which is the value suggested by the definition of nominal diameter.

(2) In the weight-diameter and strength-diameter relationships, the exponent of  $D$  is significantly less than the value 2 suggested by the simplest physical theory.

(3) In the strength-weight relationship, the exponent of  $W$  is significantly different from unity.

If "true" values of  $a$  and  $b$  may be postulated, then it follows that there exists a "true" value of the ordinate of the equation of the mean for each given value of the independent variable. Investigation of the appropriate standard errors reveals that theoretically with high probability the mean values of  $C$ ,  $W$ , and  $S$  do not deviate from the corresponding "true" values by more than 2 percent in the case of  $C$  and  $W$ , and 4 percent in the case of  $S$ . But this statement must be interpreted with much caution, not only because of the non-randomness of the sampling, but also because even if the curves had been fitted in some analogous manner to the *universe* instead of to the sample, it is possible that deviations of this order from the actual true means of the universe might be observed, because of the fact that the simple type of curve chosen for fitting may not accurately describe the real functional relation between the true means and the independent variables.

The preceding discussion of standard errors may be summarized by stating that from the viewpoint of theoretical statistical analysis, the curves have been fitted with a considerable amount of precision; but in default of exact knowledge as to the mechanism of the sampling method, the real accuracy of the curves as a description of the rational manila rope technological situation during the data period must be taken largely on faith.

The information in table 2 has been tabulated numerically in table 3, and represented graphically in figures 1, 2, 3, and 4. The "Estimates of Mean" columns in table 3 and the central curves in all of the graphs were obtained by straightfor-

ward substitution into the equations in table 2. The small circles that appear on the graphs represent the means of the observations for the respective indicated values of the independent variable.

The information as to dispersion of *individual observations* about the curve of means is presented in table 2 in the form of the standard error of estimate  $s$ . For convenience in the applications, this dispersion information is presented in table 3 in terms of tolerance limits obtained in the following manner:

Reverting to the notation of equation 2 of section III, if it be assumed that the value of  $\log Y$  given by eq 2 is the true mean of values of  $\log Y$  for each value of  $X$ , and  $s$  is the true standard deviation, then the interval  $[\log Y - ts, \log Y + ts]$  will bracket a fixed proportion  $p(t)$  of the underlying distribution, the proportion being dependent only on the value of  $t$ . (The trivially small size of all standard errors involved in the present case makes these assumptions entirely tenable from the theoretical point of view.) In the present case, the value of  $t$  was so chosen that if the distribution of individual value of  $\log Y$  were normal or Gaussian, the value of  $p(t)$  would be 0.95. Specifically, this value of  $t$  to 6 decimal places is 1.959964. Thus the formulas for the tolerance limits appearing in table 3 were:

$$\begin{aligned}\text{Lower tolerance limit} &= 10^{-1.959964s} Y \\ \text{Upper tolerance limit} &= 10^{1.959964s} Y\end{aligned}$$

In tabulating the numerical value of the tolerance limits given by these formulas, the general practice was to round off to the number of significant figures appearing in the raw data. However, a number of exceptions were made, chiefly in the direction of retaining one extra significant figure, to conform with the conventions of tabulation. Ambiguous cases were always rounded *outwards*.

The tolerance limits are plotted on the graphs as the outer curves. It should be emphasized that *these outer curves pertain to individual measurements, and not to means, such as those represented by the circles on the graphs*.

In general, the analytic representation of the 863 observations is remarkably faithful, as can be seen from a glance at the figures. (In interpreting deviations of the circles from the central line, the varying number of observations represented by

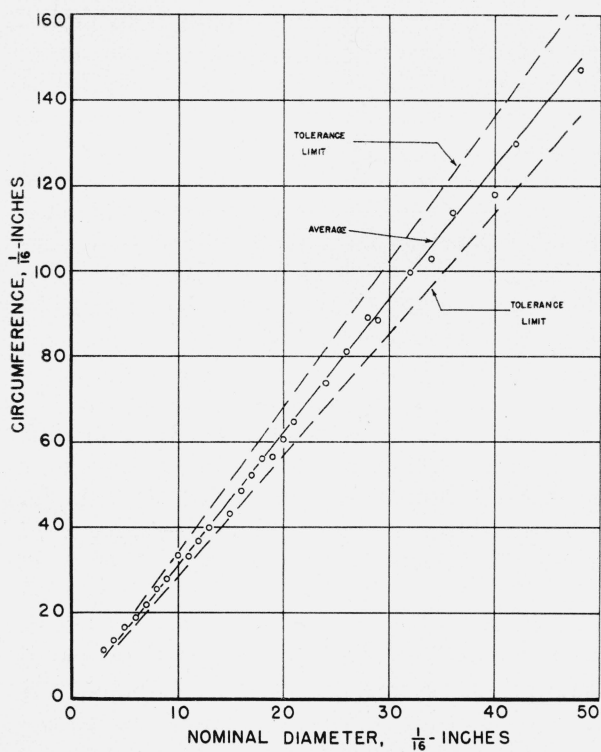


FIGURE 1.—Circumference as a function of nominal diameter

The central curve is that of the first equation in table 2.

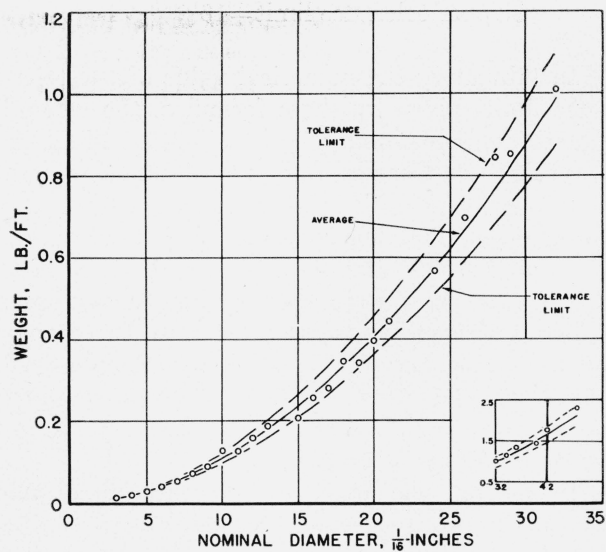


FIGURE 2.—Weight as a function of nominal diameter

The central curve is that of the second equation in table 2.

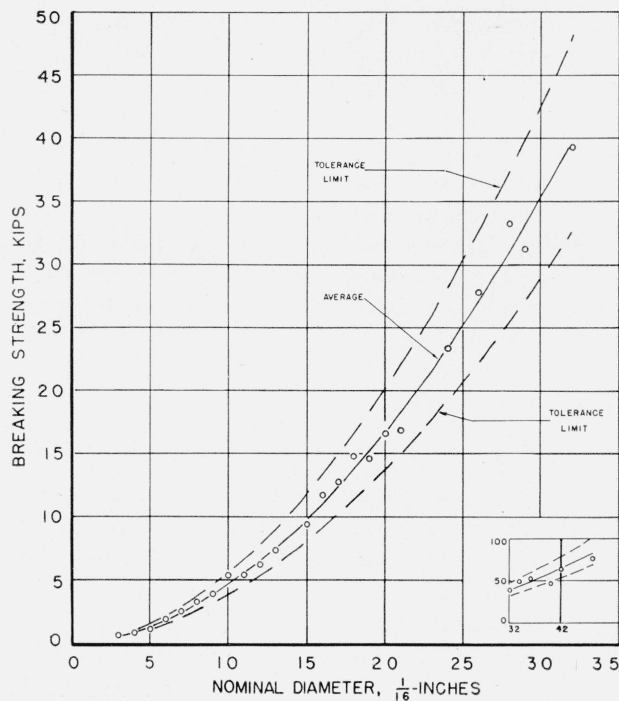


FIGURE 3.—Breaking strength as a function of nominal diameter.

The central curve is that of the third equation in table 2.

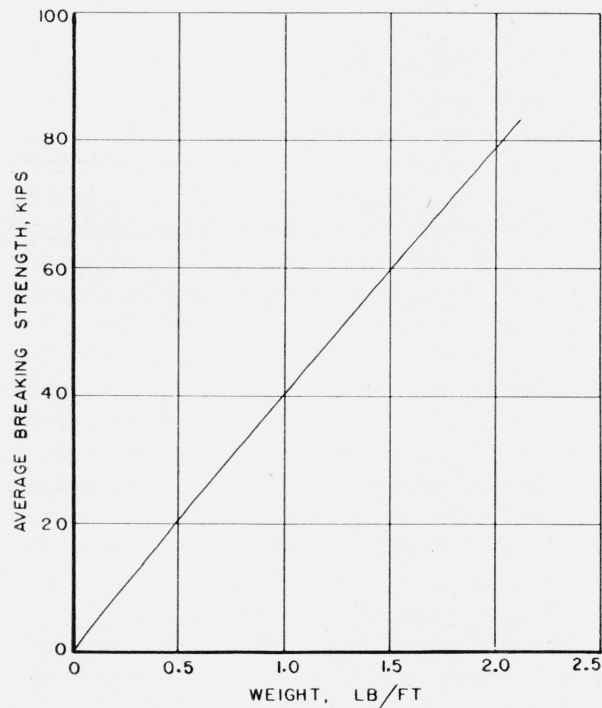


FIGURE 4.—Breaking strength as a function of weight.

The curve is that of the fourth equation in table 2.

TABLE 3.—Circumference, weight, and strength of 3-strand manila rope

Nominal diameter		Number of ropes tested	Circumference			Weight			Strength			Nominal diameter	
			Estimate of mean	Tolerance limits for individual ropes		Estimate of mean	Tolerance limits for individual ropes		Estimate of mean	Tolerance limits for individual ropes			
				$\frac{1}{16}$ in.	$\frac{1}{8}$ in.		lb/ft	lb/t		lb	lb		
in.	$\frac{1}{16}$ in.		$\frac{1}{16}$ in.	$\frac{1}{8}$ in.		lb/ft	lb/t		lb	lb		in.	$\frac{1}{16}$ in.
$\frac{3}{16}$	3	6	a 9.4	a 9	a 10	a 0.0114	a 0.010	a 0.013	525	430	630	$\frac{3}{16}$	3
$\frac{1}{4}$	4	47	12.5	11	14	.0197	.017	.022	889	730	1,070	$\frac{1}{4}$	4
$\frac{5}{16}$	5	21	15.6	14	17	.0300	.027	.034	1,336	1,100	1,610	$\frac{5}{16}$	5
$\frac{3}{8}$	6	60	18.7	17	21	.0422	.038	.047	1,865	1,530	2,250	$\frac{3}{8}$	6
$\frac{7}{16}$	7	3	21.8	20	24	.0564	.050	.063	2,472	2,030	2,980	$\frac{7}{16}$	7
$\frac{1}{2}$	8	106	25.0	23	27	.0726	.065	.081	3,156	2,590	3,810	$\frac{1}{2}$	8
$\frac{9}{16}$	9	5	28.1	26	31	.0906	.081	.101	3,914	3,210	4,720	$\frac{9}{16}$	9
$\frac{5}{8}$	10	19	31.2	28	34	.1104	.098	.124	4,745	3,900	5,720	$\frac{5}{8}$	10
$\frac{11}{16}$	11	18	34.3	31	38	.1322	.118	.148	5,648	4,640	6,810	$\frac{11}{16}$	11
$\frac{3}{4}$	12	106	37.4	34	41	.1557	.139	.174	6,622	5,440	7,990	$\frac{3}{4}$	12
$\frac{13}{16}$	13	8	40.5	37	44	.1810	.161	.203	7,666	6,290	9,250	$\frac{13}{16}$	13
$\frac{7}{8}$	14	-----	43.7	40	48	.2081	.185	.233	8,778	7,210	10,590	$\frac{7}{8}$	14
$\frac{15}{16}$	15	5	46.8	43	51	.2370	.211	.265	9,958	8,180	12,010	$\frac{15}{16}$	15
1	16	104	49.9	45	55	.2676	.238	.300	11,210	9,200	13,520	1	16
$1\frac{1}{16}$	17	4	53.0	48	58	.3000	.267	.336	12,520	10,280	15,100	$1\frac{1}{16}$	17
$1\frac{1}{8}$	18	8	56.1	51	61	.3340	.297	.374	13,900	11,410	16,770	$1\frac{1}{8}$	18
$1\frac{1}{4}$	19	11	59.3	54	65	.3698	.329	.414	15,340	12,600	18,500	$1\frac{1}{4}$	19
$1\frac{1}{2}$	20	112	62.4	57	68	.4073	.363	.456	16,850	13,830	20,330	$1\frac{1}{2}$	20
$1\frac{3}{8}$	21	7	65.5	60	72	.4465	.397	.500	18,420	15,120	22,220	$1\frac{3}{8}$	21
$1\frac{1}{2}$	22	-----	68.6	62	75	.4874	.434	.546	20,060	16,470	24,200	$1\frac{1}{2}$	22
$1\frac{7}{8}$	23	-----	71.7	65	79	.5299	.472	.593	21,760	17,870	26,250	$1\frac{7}{8}$	23
$1\frac{1}{2}$	24	110	74.9	68	82	.5741	.511	.643	23,520	19,310	28,370	$1\frac{1}{2}$	24
$1\frac{9}{16}$	25	-----	78.0	71	85	.6200	.552	.694	25,340	20,810	30,570	$1\frac{9}{16}$	25
$1\frac{5}{8}$	26	10	81.1	74	89	.6675	.594	.747	27,220	22,350	32,840	$1\frac{5}{8}$	26
$1\frac{1}{2}$	27	-----	84.2	77	92	.7167	.638	.802	29,170	23,950	35,190	$1\frac{1}{2}$	27
$1\frac{3}{4}$	28	9	87.3	80	96	.7675	.683	.859	31,170	25,590	37,600	$1\frac{3}{4}$	28
$1\frac{13}{16}$	29	13	90.4	82	99	.8193	.729	.917	33,240	27,290	40,100	$1\frac{13}{16}$	29
$1\frac{7}{8}$	30	-----	93.6	85	103	.8739	.778	.978	35,360	29,030	42,650	$1\frac{7}{8}$	30
$1\frac{15}{16}$	31	-----	96.7	88	106	.9295	.827	1.041	37,550	30,830	45,300	$1\frac{15}{16}$	31
2	32	50	99.8	91	109	.9868	.878	1.105	39,790	32,670	48,000	2	32
$2\frac{1}{16}$	33	-----	102.9	94	113	1.0456	.931	1.171	42,090	34,560	50,770	$2\frac{1}{16}$	33
$2\frac{1}{8}$	34	1	106.0	97	116	1.1061	.985	1.238	44,450	36,500	53,620	$2\frac{1}{8}$	34
$2\frac{3}{16}$	35	-----	109.2	99	120	1.1682	1.040	1.308	46,870	38,480	56,540	$2\frac{3}{16}$	35
$2\frac{1}{4}$	36	13	112.3	102	123	1.2318	1.097	1.379	49,350	40,520	59,530	$2\frac{1}{4}$	36
$2\frac{5}{16}$	37	-----	115.4	105	126	1.2970	1.154	1.452	51,890	42,600	62,590	$2\frac{5}{16}$	37
$2\frac{3}{8}$	38	-----	118.5	108	130	1.3669	1.217	1.530	54,480	44,730	65,720	$2\frac{3}{8}$	38
$2\frac{7}{16}$	39	-----	121.6	111	133	1.4321	1.275	1.603	57,130	46,910	68,920	$2\frac{7}{16}$	39
$2\frac{1}{2}$	40	1	124.8	114	137	1.5020	1.337	1.682	59,830	49,120	72,170	$2\frac{1}{2}$	40
$2\frac{9}{16}$	41	-----	127.9	117	140	1.5735	1.401	1.762	62,600	51,400	75,510	$2\frac{9}{16}$	41
$2\frac{5}{8}$	42	4	131.0	119	143	1.6466	1.466	1.844	65,420	53,710	78,920	$2\frac{5}{8}$	42
$2\frac{11}{16}$	43	-----	134.1	122	147	1.7211	1.532	1.927	68,290	56,070	82,380	$2\frac{11}{16}$	43
$2\frac{3}{4}$	44	-----	137.2	125	150	1.7972	1.600	2.012	71,220	58,480	85,910	$2\frac{3}{4}$	44
$2\frac{13}{16}$	45	-----	140.4	128	154	1.8749	1.669	2.099	74,210	60,930	89,520	$2\frac{13}{16}$	45
$2\frac{7}{8}$	46	-----	143.5	131	157	1.9542	1.739	2.188	77,250	63,430	93,190	$2\frac{7}{8}$	46
$2\frac{15}{16}$	47	-----	146.6	134	161	2.0349	1.811	2.278	80,350	65,970	96,930	$2\frac{15}{16}$	47
3	48	2	149.7	136	164	2.1172	1.885	2.370	83,500	68,560	100,700	3	48

a See table 4.

each circle must be taken into account.) The representation of dispersion is apparently as successful as that of the mean values. As a partial check it might be noted that, for example, 34 observations on  $S$  fell outside the tolerance limits for  $S$  given in table 3; this is 4 percent of the total number of observations on  $S$ , a proportion that compares favorably with the specified theoretical value of 5 percent. The fact that the number of observations that were too high was about equal to the number out on the low side gives added credence to the validity of the logarithmic transformation. Similar situations obtain in the cases of  $W$  and  $C$ .

The following irregularities in the data should be explicitly noted:

(1) The observed distributions of  $C$ ,  $W$ , and  $S$  for nominal diameters of  $\frac{5}{16}$  in.,  $1\frac{1}{8}$  in., and  $1\frac{3}{4}$  in., not found in the Federal Specification [2], were very similar to the corresponding distributions for the nominal diameters of  $1\frac{1}{16}$  in.,  $1\frac{3}{16}$  in., and  $1\frac{13}{16}$  in., respectively. In each case, the means and tolerance limits given in table 2 for the larger size represented the mean and range of the observation for the smaller size (as well as for the larger size) much better than the mean and tolerance limits given in table 2 for the smaller size. Apparently, rope of the appropriate one of the three larger sizes is supplied whenever one of the three smaller sizes is stipulated in a purchase.

(2) The observed values of  $C$ ,  $W$ , and  $S$  for a fixed value of  $D$  exhibited some tendency to occur in clusters having smaller individual dispersion than that represented by the values of  $s$  in table 2 or the tolerance limits in table 3. This phenomenon was undoubtedly due in part to non-random sampling.

The chief result of misclassification such as noted in (1) above, and of the clustering noted in (2), is to increase the values of  $s$  (and thus increase the spread of the tolerance limits) over the values that would have been obtained if such irregularities had been absent.

(3) In the case of the  $\frac{3}{16}$  in. nominal diameter, the observed circumference and weights of the six ropes tested were not properly represented by the analytical expression. In view of the various testing, rounding off, sampling, and classification errors involved in the measurements, it may well

be that extrapolation from the analytic representation based on over 800 measurements should furnish more reliable information than a sample of six possibly anomalous observations. However, for completeness, the means and ranges of the  $C$  and  $W$  observations for  $D=\frac{3}{16}$  are given in table 4.

TABLE 4.—Summary of observed circumferences and weights for  $\frac{3}{16}$  in. 3-strand manila rope

[Based on 6 observations on each variable]

	Circumference	Weight
	$\frac{3}{16}$ in.	lb/ft
Arithmetic mean.....	11.2	0.0142
Minimum.....	10	.013
Maximum.....	12	.015

Figure 5 exhibits a comparison between the results on mean strength obtained in the present paper, and the curve fitted by Stang and Strickenberg [1] to the data on strength which they obtained in 1921. An examination of the closeness of fit of Stang and Strickenberg's curve and of the

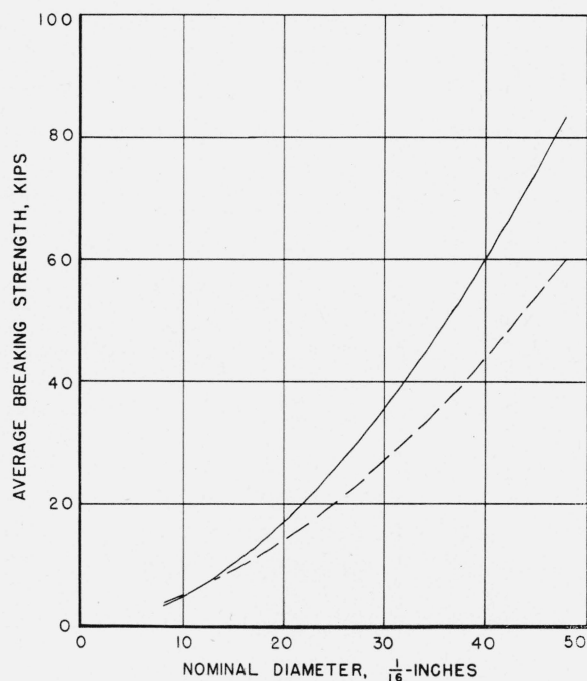


FIGURE 5.—Comparison of present sample with that of reference [1]

The solid line is the present sample. The broken line is the sample of reference [1]



dispersion of their data, indicates that the theoretical accuracy of their curve should be roughly comparable with that of the corresponding curve in the present paper when due allowance is made for the fact that their curve is based on only about one-half as many observations. It follows that for sizes of 1 in. diameter and greater, the average breaking strength of rope in 1921 as represented by Stang and Strickenberg's curve is significantly lower than that of the rope discussed in the present paper.

## V. References

- [1] A. H. Stang and L. R. Strickenberg, Techn. Pap. BS (1921) T198.
- [2] Federal Standard Stock Catalog Section IV (Part 5), Federal Specification for Rope; Manila, T-R-601a, and amendments 1 and 2 (November 26, 1935).
- [3] H. L. Whittemore, Commercial Standards Monthly 8, No. 2, 57 (1931).
- [4] Unpublished data.
- [5] J. H. Curtiss, Annals of Math. Statistics **14**, 107 (1943).

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